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ON THE THEORY OF THICK SLABS ON AN ELASTIC FOUNDATION*

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Static problems are investigated covering the deformation of a thick elastic slab with an arbitrary smooth boundary that rests without friction on a linearly deformable foundation. The slab is loaded by a normal distributed load on the upper face. Lur'e's symbolic method is used to investigate the state of stress and strain of the slab. A class of homogeneous solutions corresppondlng to a free upper face and the condition of linearly elastic contact of the slab with the foundation on the lower face of the slab is isolated. This last condition is a relation realized by the integral operator of the contact problem for a linearly deformable foundation between the settling of the slab and the contact stress. It is shown that the homogeneous solution determined in this manner can be of three kinds: potential, vortical, and harmonic. There is also a certain elementary solution. The problem here of finding the characteristic numbers of the potential solution reduces to seeking the eigenvalues of the integral operator mentioned. The axisymmetric problem of the deformation of a thick circular slab in a Winkler foundation is considered as an example.

The problem under consideration was extensively investigated earlier within the framework of the applied theories of slabs: Kirchhoff-Love /1—3/, Reissner /4, 5/, and others /5/.

1. Consider a slab of isotropic elastic material resting without friction on a linearly deformable isotropic foundation. We take the slab middle plane as the plane x_1, x_2 of a rectangular Cartesian coordinate system and denote the slab thickness by $2h$. Let Γ be the cylindrical boundary of the slab.

Investigation of the slab state of stress and strain reduces to solving the Lamé equilibrium equations in the displacement $u_i = \{u, v, w\}$ /6/. The stresses σ_{ij} are expressed in terms of the displacements by using the generalized Hooke's law /6/. For simplicity, we limit ourselves to the case when a normal distributed load is applied to the slab upper face. Then the boundary conditions on the endfaces have the form

$$\begin{aligned} &z = h, \quad \sigma_{3\alpha} = 0 \quad (\alpha = 1, 2), \quad {}^1/_2\sigma_{33}/G = p(x_1, x_2) \\ &z = -h, \quad \sigma_{3\alpha} = 0 \quad (\alpha = 1, 2), \quad u_3 = L\sigma_{33} \end{aligned} \tag{1.1}$$

Here L is a linear operator expressing the relationship between the settling of the foundation surface and the normal load acting on it, where /3, 7 /

$$Lf(x_1, x_2) = \iint_S K(x_1 - u_1, x_2 - u_2) f(u_1, u_2)\, du_1\, du_2 \tag{1.2}$$

$$K(x_1, x_2) = \iint_{-\infty}^{\infty} L(R) \exp[-i(\xi x_1 + \eta x_2)]\, d\xi\, d\eta, \quad R = \sqrt{\xi^2 + \eta^2}$$

(S is the domain occupied by the slab). The function $L(R)$ possesses certain special properties /3, 7/. The kernel $K(x_1, x_2)$ is positive and symmetric in the domain S.

If it is a Winkler base, then

$$Lf = f/c \tag{1.3}$$

(c is the bed coefficient), which corresponds to $L(R) = \text{const}$.

The boundary conditions on the slab lateral surface are specified by the acting load given in L.

We construct the solution of the problem by using Lur'e's symbolic method. The solution of the Lamé equation can be obtained in the form /6, 8/

$$\left\| \begin{matrix} u \\ v \end{matrix} \right\| = \cos zD \left\| \begin{matrix} u_0 \\ v_0 \end{matrix} \right\| - \frac{\varkappa}{2} \frac{z \sin zD}{D} \partial_\alpha (\partial_1 u_0 + \partial_2 v_0 + w_0') + \tag{1.4}$$

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$$\frac{\sin zD}{D}\begin{Vmatrix}u_0'\\ v_0'\end{Vmatrix} - \frac{\varkappa}{2(\varkappa+1)}\left(\frac{\sin zD}{D^3} - \frac{z\cos zD}{D^2}\right)\partial_\alpha(\partial_1 u_0' + \partial_2 v_0' - D^2 w_0), \quad \alpha = \begin{Vmatrix}1\\ 2\end{Vmatrix}$$

$$w = \frac{\sin zD}{D} w_0' + \frac{\varkappa}{2}\left(\frac{\sin zD}{D} - z\cos zD\right)(\partial_1 u_0 + \partial_2 v_0 + w_0') + \cos zD w_0 - \frac{\varkappa}{2(\varkappa+1)}\frac{z\sin zD}{D}(\partial_1 u_0' + \partial_2 v_0' - D^2 w_0)$$

$$\varkappa = 1/(1-2\nu), \qquad D^2 = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2$$

Here ν is Poisson's ratio, and G is the slab shear modulus. The stresses can be found from(1.4) on the basis of Hooke's law and have a similar form.

To determine the six unknown functions u_0, v_0, w_0' and u_0', v_0', w_0 dependent on x_1, x_2 we use the boundary conditions on the facial surfaces (1.1). We write these conditions in the form

$$\frac{\sigma_{\alpha 3}(z=h) + \sigma_{\alpha 3}(z=-h)}{2G} = \cos hD\left(\begin{Vmatrix}u_0'\\ v_0'\end{Vmatrix} + \partial_\alpha w_0\right) - \frac{\varkappa}{\varkappa+1}\frac{h\sin hD}{D}\partial_\alpha(\partial_1 u_0' + \partial_2 v_0' - D^2 w_0) = 0 \tag{1.5}$$

$$\frac{-\sigma_{\alpha 3}(z=h) + \sigma_{\alpha 3}(z=-h)}{2G} = \varkappa h\cos hD\partial_\alpha(\partial_1 u_0 + \partial_2 v_0 + w_0') + \frac{\sin hD}{D}\left(D^2\begin{Vmatrix}u_0\\ v_0\end{Vmatrix} - \partial_\alpha w_0'\right) = 0$$

$$\frac{\sigma_{33}(z=h)}{2G} = -\frac{1}{\varkappa+1}\left[\varkappa h\cos hD(\partial_1 u_0' + \partial_2 v_0' - D^2 w_0) + \frac{\sin hD}{D}(\partial_1 u_0' + \partial_2 v_0') + (2\varkappa+1)D\sin hD w_0\right] + \varkappa hD\sin hD(\partial_1 u_0 + \partial_2 v_0 + w_0') + (\varkappa-1)\cos hD(\partial_1 u_0 + \partial_2 v_0) + (\varkappa+1)\cos hD w_0' = p(x_1, x_2) \tag{1.6}$$

$$GL\left\{\frac{1}{\varkappa+1}\left[\frac{\sin hD}{D}(\partial_1 u_0' + \partial_2 v_0') + (2\varkappa+1)D\sin hD w_0 + \varkappa h\cos hD(\partial_1 u_0' + \partial_2 v_0' - D^2 w_0)\right] + \varkappa hD\sin hD(\partial_1 u_0 + \partial_2 v_0 + w_0') + (\varkappa-1)\cos hD(\partial_1 u_0 + \partial_2 v_0) + (\varkappa+1)\cos hD w_0'\right\} = \cos hD w_0 - \frac{\varkappa}{2(\varkappa+1)}\frac{h\sin hD}{D}(\partial_1 u_0' + \partial_2 v_0' - D^2 u_0) - \frac{\sin hD}{D}w_0' - \frac{\varkappa}{2}\left(\frac{\sin hD}{D} - h\cos hD\right)(\partial_1 u_0 + \partial_2 v_0 + w_0') \tag{1.7}$$

To solve system (1.5)-(1.7), we use the method developed in /6, 8/. We set

$$u_0' = L_1\partial_1 P_1 + \partial_2 V_1, \quad v_0' = L_1\partial_2 P_1 - \partial_1 V_1, \quad w_0 = L_2 P_1$$
$$u_0 = L_3\partial_1 P_2 + \partial_2 V_2, \quad v_0 = L_3\partial_2 P_2 - \partial_1 V_2, \quad w_0' = L_4 P_2 \tag{1.8}$$

where P_1, P_2, V_1, V_2 are new unknown functions, and L_1, L_2, L_3, L_4 are certain differential operators of infinitely high order. The functions P_1, P_2 determine the potential solution and the functions V_1, V_2 the vortex solution.

2. By virtue of the linearity of the problem, a separate construction of the potential and the vortex solutions is possible. We first study the potential solution in detail. It is seen that (1.5) are equivalent to the following two equations in the functions P_1 and P_2:

$$\left[\left(\cos hD - \frac{\varkappa}{\varkappa+1}hD\sin hD\right)L_1 + \left(\cos hD + \frac{\varkappa}{\varkappa+1}hD\sin hD\right)L_2\right]P_1 = 0 \tag{2.1}$$

$$\left[(\varkappa hD^2\cos hD + D\sin hD)L_3 + \left(\varkappa h\cos hD - \frac{\sin hD}{D}\right)L_4\right]P_2 = 0$$

These equations will be satisfied identically if we set

$$L_1 = \cos hD + \frac{\varkappa}{\varkappa+1}hD\sin hD \tag{2.2}$$

$$L_2 = -\left(\cos hD - \frac{\varkappa}{\varkappa+1}hD\sin hD\right)$$

$$L_3 = \varkappa\cos hD - \frac{\sin hD}{hD}, \quad L_4 = -\left(\varkappa D^2\cos hD + \frac{D}{h}\sin hD\right)$$

Taking (2.2) into account, we obtain equations in P_1 and P_2 after substituting (1.8) into the remaining two Eqs.(1.6) and (1.7)

$$hD(2hD-\sin 2hD)P_1+(\varkappa+1)D(2hD+\sin 2hD)P_2= -2h\frac{\varkappa+1}{\varkappa}p(x_1,x_2) \tag{2.3}$$
$$\left[\frac{\varkappa G}{\varkappa+1}LD(2hD-\sin 2hD)+\cos^2 hD\right]P_1- \left[\frac{\varkappa G}{h}LD(2hD+\sin 2hD)+\frac{\varkappa+1}{h}\sin^2 hD\right]P_2=0$$

The solution of system (2.3) can be represented as the sum of some particular solution of this system and a general solution of the homogeneous system corresponding to the case $p(x_1,x_2)=0$. The problem of constructing the particular solution can obviously be reduced to the following problem for an infinite layer.

We consider the equilibrium of an infinite layer of thickness $2h$ whose middle plane coincides with the x_1, x_2 plane under the following boundary conditions:

$$z=h,\quad \sigma_{3\alpha}=0\,(\alpha=1,2),\ \frac{\sigma_{33}}{2G}=\begin{cases}p, & (x_1,x_2)\in S\\ 0, & (x_1,x_2)\overline{\in} S\end{cases} \tag{2.4}$$

(the continuation to zero is optional)

$$z=-h,\ \sigma_{3\alpha}=0\quad(\alpha=1,2),\quad u_3=L\sigma_{33},\quad (x_1,x_2)\in R_2$$

The last equation denotes the extention of the contact condition in the interior of the domain S, i.e., the application of the last relationship in (1.1) for all x_1, x_2.

We will obtain the solution of problem (2.4) by first considering a given function u_3 for $z=-h$. By applying a two-dimensional Fourier transform in the variables x_1, x_2 we then obtain that for $z=-h$

$$\frac{\Sigma_{33}}{2G}=PK_1(\alpha h)-\frac{U_3}{h}K_2(\alpha h),\quad \alpha=\sqrt{\alpha_1^2+\alpha_2^2} \tag{2.5}$$
$$K_1(u)=2\frac{\operatorname{sh}2u+2u\operatorname{ch}2u}{\operatorname{sh}4u+4u},\quad K_2(u)=\frac{8\varkappa}{\varkappa+1}u\frac{\operatorname{sh}^2 2u-4u^2}{\operatorname{sh}4u+4u}$$

The capital letters here denote the Fourier transforms of the corresponding functions that depend on the variables α_1, α_2. We now take into account that the last boundary condition for $z=-h$ actually has the form (2.4), therefore

$$U_3=L(\alpha h)\iint_S \sigma_{33}(x_1,x_2)\exp[i(\alpha_1x_1+\alpha_2x_2)]\,dx_1\,dx_2$$

Substituting this relationship into (2.5) and performing an inversion therein, we obtain an equation for the function σ_{33} for $z=-h$

$$\frac{\sigma_{33}}{2G}=f-\iint_{S}\sigma_{33}(u_1,u_2)\,du_1\,du_2\iint_{-\infty}^{\infty}L(\alpha)K_2(\alpha)\times \exp\{-i[\alpha_1(x_1-u_1)+\alpha_2(x_2-u_2)]\}\,dx_1\,dx_2,\quad (x_1,x_2)\in S \tag{2.6}$$

($f(x_1,x_2)$ is the original of the function $P(\alpha_1,\alpha_2)K_1(\alpha)$).

Thus, in the general case the problem of finding the particular solution of the inhomogeneous problem is successfully reduced to a two-dimensional Fredholm integral equation of the second kind.

We will now consider the construction of the homogeneous potential solution. We call the solution corresponding to a free upper endface and a contact condition with the base at the lower endface the homogeneous solution of the problem under consideration. Let $p(x_1,x_2)=0$, then the first equation of (2.3) is satisfied identically if the stress function Φ is introduced

$$P_1=(\varkappa+1)D(2hD+\sin 2hD)\Phi,\quad P_2=-hD(2hD-\sin 2hD)\Phi \tag{2.7}$$

The second equation of (2.3) therefore takes the form

$$\left[\frac{G}{h}\frac{\varkappa}{\varkappa+1}LD^2(4h^2D^2-\sin^2 2hD)+D^2\left(1+\frac{\sin 4hD}{4hD}\right)\right]\Phi=0 \tag{2.8}$$

As in the classical theory of slabs /6, 8/, we will seek the solution of (2.8) in the class of metaharmonic functions in the domain S

$$(D^2-\gamma^2/h^2)\Phi=0 \tag{2.9}$$

We then obtain the following relationship from (2.8)

$$\gamma^2L\Phi=\gamma^2\mu\Phi \tag{2.10}$$

where

$$\mu = -\left(1 + \frac{\sin 4\gamma}{4\gamma}\right)\left[\frac{G}{h}\frac{\varkappa}{\varkappa+1}(4\gamma^2 - \sin^2 2\gamma)\right]^{-1} \tag{2.11}$$

A solution different from zero for (2.10) exists in the class of operators (1.1), (1.2) with symmetric positive kernel only provided that $\mu = \mu_n$ is an eigenvalue of the operator L, while Φ_n is its corresponding eigenfunction ($n = 1, 2, \ldots$) /9/. The complex characteristic numbers γ_{nk} ($\operatorname{Re}\gamma_{nk} > 0$), $n, k = 1, 2, \ldots$ are found from (2.11) with the values $\mu_n > 0$. However, the eigenfunctions Φ_n of the operator L are known not to satisfy Eq.(2.9) in the general case; consequently the function Φ_n should be represented in the form of the expansion

$$\Phi_n = \sum_{k=1}^{\infty} a_{nk}\Phi_{nk}, \quad n = 1, 2, \ldots \tag{2.12}$$

in the functions Φ_{nk}, which is a solution of (2.9) for $\gamma = \gamma_{nk}$. The qeustions of completeness that emerge here require further analysis.

The value $\gamma = 0$ is known to satisfy (2.10), and as is seen from (2.9), corresponds to the harmonic solution. Exactly as in classical theory /6, 8/, the biharmonic solution is separated from the potential homogeneous solution, and the harmonic solution is separated out in a natural manner. The stresses and displacements corresponding to the harmonic solution have the form

$$u_\alpha = \partial_\alpha\Phi \quad (\alpha = 1, 2), \quad u_3 = 0 \tag{2.13}$$

$$\sigma_{3\alpha} = \sigma_{33} = 0, \quad \frac{\sigma_{\alpha\beta}}{2G} = \partial_\alpha\partial_\beta\Phi \quad (\alpha, \beta = 1, 2), \quad D^2\Phi = 0$$

The absence here of the quadruple root $\gamma = 0$ corresponding to the biharmonic solution of classical theory is due to the fact that the two zero roots γ are, in effect, transformed into roots of the characteristic Eq.(2.11).

We will now consider the construction of the vortex solution. Subsituting the vortex part of the relationships (1.8) into (1.5)-(1.7) (for $p(x_1, x_2) = 0$), we note that (1.6) and (1.7) are satisfied identically here. The remaining relationships (1.5) yield

$$\partial_\alpha \cos hDV_1 = 0, \quad \partial_\alpha D \sin hDV_2 = 0 \quad (\alpha = 1, 2) \tag{2.14}$$

Let the functions V_{1k} and V_{2k} satisfy the metaharmonic equations

$$(D^2 - \sigma_k^2/h^2)\, V_{1k} = 0, \quad (D^2 - \delta_k^2/h^2)\, V_{2k} = 0 \tag{2.15}$$

We then obtain from (2.14)

$$\sigma_k = \pi k - \pi/2, \quad \delta_k = \pi k \quad (k = 1, 2, \ldots) \tag{2.16}$$

The displacement vector components for the vortex part of the solution have the form

$$w = 0 \tag{2.17}$$

$$u_\alpha = (-1)^\beta 2\varkappa h\left[h\sum_{k=1}^{\infty}\frac{\sin\sigma_k\zeta}{\sigma_k}\partial_\beta V_{1k} + \sum_{k=1}^{\infty}\cos\delta_k\zeta\,\partial_\beta V_{2k}\right]$$

$$\zeta = \frac{z}{h}$$

An elementary homogeneous solution that leaves the foundation undeformed

$$\left\|\begin{matrix} u \\ v \end{matrix}\right\| = -\frac{1-\nu}{2\nu}x_\alpha, \quad \alpha = \left\|\begin{matrix} 1 \\ 2 \end{matrix}\right\|, \quad w = z + h \tag{2.18}$$

$$\sigma_{\alpha\alpha} = -G\frac{1+\nu}{\nu}, \quad \sigma_{33} = \sigma_{3\alpha} = \sigma_{12} = 0, \quad \alpha = 1, 2$$

still exists in addition to the solutions constructed above for the problem in question.

3. The expression for the operator L (1.2) is usually known explicitly for a specific kind of linearly deformable base. For instance, if the base is an elastic half-space then

$$L(R) = \frac{1-\nu_1}{4\pi^2 G_1}\frac{1}{R} \tag{3.1}$$

If the base foundation is an infinite layer resting without friction on a rigid base then

$$L(R) = \frac{1-\nu_1}{4\pi^2 G_1}\frac{\operatorname{ch}2R - 1}{(\operatorname{sh}2R + 2R)R} \tag{3.2}$$

If the layer adheres completely to the underlying rigid base, then

$$L(R) = \frac{1-\nu_1}{4\pi^2 G_1}\frac{2\varkappa_1\operatorname{sh}2R - 4R}{(2\varkappa_1\operatorname{ch}2R + 1 + \varkappa_1^2 + 4R^2)R}, \quad \varkappa_1 = 3 - 4\nu_1 \tag{3.3}$$

(G_1, ν_1 are the elastic constants of the base). Other kinds of linearly deformable bases are known /7/.

For such a base the potential part of the solution is constructed in conformity with the theory elucidated in Sects.1, 2.

A different situation arises if the base is a Winkler foundation (1.3). Then the eigenvalue $\mu = 1/c$ of the operator L (2.10) is unique, and for these values (2.10) is satisfied identically for an arbitrary function Φ. Therefore, in this case the potential homogeneous solutions Φ_n should be found from (2.9) taken for $\gamma = \gamma_n$, where γ_n ($\operatorname{Re}\gamma_n > 0$) are complex roots of the transcendental characteristic equation

$$F(\gamma) = 1 + \frac{\sin 4\gamma}{4\gamma} + A(4\gamma^2 - \sin^2 2\gamma) = 0, \quad A = \frac{G}{ch}\frac{\varkappa}{\varkappa + 1} \tag{3.4}$$

obtained from (2.11) for $\mu = 1/c$. Therefore, here as in the classical case, the vector of the characteristic numbers $\{\gamma_k\}$ is one-dimensional. The relationship (3.4) was obtained in /5/ for the two-dimensional problem about the plane state of stress of a thick beam.

In the case of the Winker foundation the particular solution of the inhomogeneous problem can also be constructed much more effectively. The fact is that here the continuation to the exterior of the domain S (the last condition in (2.4)) for the relation $u_3 = \sigma_{33}/c$ corresponds to the problem of an infinite layer loaded from above by normal forces $p(x_1, x_2)$ and resting without friction on the Winkler foundation. The solution of this problem is obtained easily by using Fourier transforms and has been studied in some detail /10/.

We note that the fundamental difficulty in solving the problems under consideration is associated with satisfying the boundary conditions on the slab lateral surface.

4. As an illustration, we consider the axisymmetric problem of the deformation of a circular slab with a free lateral surface on a Winkler foundation. Let the applied load be $p(r) = J_0(\delta r), \delta > 0$. Then the particular solution of the problem (an infinite layer on a Winkler foundation) is expressed in elementary form. We have for the Love function

$$\begin{aligned}\chi(r) = N\,[&A_1(\delta h)(\delta z \operatorname{sh}\delta h \operatorname{ch}\delta z - \delta h \operatorname{ch}\delta h \operatorname{sh}\delta z - 2\nu \operatorname{sh}\delta h \operatorname{sh}\delta z) + \\ &A_2(\delta h)(\delta z \operatorname{ch}\delta h \operatorname{sh}\delta z - \delta h \operatorname{sh}\delta h \operatorname{ch}\delta z - 2\nu \operatorname{ch}\delta h \operatorname{ch}\delta z)]\, J_0(\delta r)\end{aligned} \tag{4.1}$$

$$A_1(x) = \frac{A}{2}(\operatorname{sh}2x - 2x) + \frac{\operatorname{ch}^2 x}{2x}$$

$$A_2(x) = \frac{A}{2}(\operatorname{sh}2x + 2x) + \frac{\operatorname{sh}^2 x}{2x}, \quad N = \frac{2}{\delta^3 F(i\delta h)}$$

It can be shown that two kinds of homogeneous solutions, the harmonic and the vortex one, do not occur in the problem under consideration. To construct the homogeneous potential solution we investigate the roots of (3.4). The following asymptotic formula for values of γ_n large in absolute value that lie in the first quadrant

$$\gamma_n \underset{n\to\infty}{\sim} \left[\frac{\pi n}{2} - \frac{\pi}{4} - \frac{\ln 2\pi n}{2\pi n} + \frac{1}{4A\pi n}\right] + i\left[\frac{1}{2}\ln 2\pi n - \frac{1}{4n}\right] + O\left(\frac{\ln^2 n}{n^2}\right), \tag{4.2}$$

can be obtained by the usual methods.

Exact values of γ_n were sought by Newton's method, where its asymptotic value (4.2) was taken as the initial magnitude of the appropriate root. Since formula (4.2) loses its effectiveness for small A, as is easily seen, the component $1/(4A\pi n)$ was discarded for $A < 0.15$ in specific calculations. In such an approach the process of finding the numbers γ_n always converged (computations were performed in the range $0.01 \leqslant A \leqslant 100$).

The solution of the metaharmonic Eq.(2.9), bounded at the origin, has the following form in the axisymmetric case, as is well-known

$$\Phi_n = C_n I_0(\gamma_n h^{-1} r) \tag{4.3}$$

Further solution of the problem consists in seeking the coefficients C_n by satisfying the boundary conditions on the slab lateral surface. To do this, the general solution in the form of the sum of the inhomogeneous solution (4.1) found above, the elementary solution (2.18), and the eigenfunction series of the homogeneous problem (4.3) is written down for the stresses σ_r and τ_{rz} (the corresponding formulas are not presented because of their complexity). The boundary conditions on Γ can be satisfied by several methods. In this paper the Lagrange variational principle /11/ was used, which helped to reduce the problem to a certain linear algebraic infinite system. The following parameter values were taken: $\nu = 0.2$ (concrete) and $\delta = 2/a$ (a is the slab radius). The solutions of tenth and twentieth order systems are practically identical in all the cases considered. Consequently, we limited ourselves to 10 terms in all the series (this corresponds to five characteristic numbers γ_n and five $\bar{\gamma}_n$). The error in satisfying the boundary conditions, including on the slab edge, did not exceed 3×10^{-3} in this approach (the characteristic value of the stress is due to the selection of the applied load and equals $p(0) = 1$). The time to calculate any of the stresses or displacements at an arbitrary point of the slab is 1 sec on average on an ES-1022 computer.

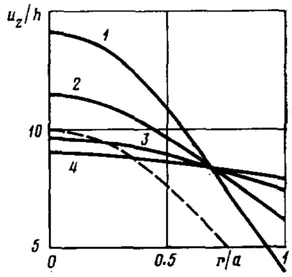

Fig.1

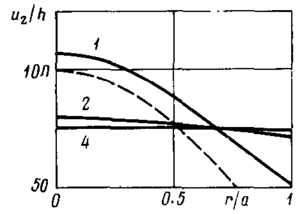

Fig.2

Curves of the slab settling (for the face $z = h$) are displayed in Figs.1 and 2 for $A = 4$ and $A = 40$ and different $\lambda = h/a$. Curves *1-4* correspond to $\lambda = 0.1, 0.2, 0.3, 0.4$. The curve of the external laod in appropriate dimensionless variables is shown for comparison by dashes. It is seen that a thin slab almost duplicates the shape of the outer load. As the slab thickness increases it is deformed less and less and for $\lambda = 0.4$ settles amost as a rigid stamp. This tendency appears more strongly on a soft base ($A = 40$) than on a stiff one ($A = 4$).

Calculations showed that domains of high negative normal stresses, exceeding the characteristic stress severalfold, can appear in thin slabs. This can result in the appearance of cracks and raptures in reinforced concrete foundations. The method in this paper enables the minimum slab thickness for which negative stresses will not exceed the allowable ones to be estimated.

In conclusion we note that the approach developed in this paper can be carried over to dynamic problems on the harmonic vibrations of a thick slab on an elastic foundation.

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